

THE SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS

BY

ARMAS LAUPA

B.C.E., Georgia Institute of Technology, 1951

M.S., University of Illinois, 1952



AN ABSTRACT OF A THESIS

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INTRODUCTION

The object of this thesis was to review and correlate the results of previous research in the field of shear and diagonal tension, to determine the modes and characteristics of shear failure of reinforced concrete beams, and to establish a general expression for the shear strength of reinforced concrete beams under different loading conditions.

The investigation was limited to beams subjected to combinations of shear and flexure. More than one thousand tests of such beams with a wide range of physical properties and under different types of loading were studied.

SIMPLE-SPAN RECTANGULAR BEAMS UNDER ONE OR TWO SYMMETRICAL CONCENTRATED LOADS

A basic expression for the shear strength of reinforced concrete beams was derived by considering simple-span rectangular beams without web reinforcement and subjected to one or two symmetrical concentrated loads. It was first assumed that the total shear force was resisted solely by the compression area of the concrete and that the criterion of failure was an ultimate average shearing stress, related to the compressive strength of the concrete. These assumptions yielded an expression in a form which suggested that the real criterion for shear failures was a limiting moment rather than an ultimate shearing stress and that shear failures were actually a compression phenomenon.

An empirical expression for the ultimate shear-compression moment was established by treating shear failures as compression failures and assuming that the depth of the compression zone of the concrete was related to k as determined by the elastic "straight line" theory. This expression was based on the test results from 15 different investigations involving 111 beams which failed in shear. These beams were tested over a period of 43 years and had a wide variation in their physical properties. The average ratio of measured to computed moments was 0.993 and the standard deviation 0.120.

This basic equation for the shear-compression moment was interpreted theoretically in the light of the conventional theory of compression failures of reinforced concrete beams. It was concluded that the criterion of failure was still the ultimate compressive strain of the concrete. Contrary to flexural compression failures, however, the strain distribution over the depth of the beam was not linear at the section of failure. Furthermore, there must be a certain concentration of concrete strain in the region of the diagonal crack. Since the actual distribution of strain could not be determined, no theoretical relationship could be written for the depth of the compression zone, $k_s d$, and the ultimate concrete strain at shear failure. In order to write a general expression for shear-compression failures, either the value of k_s , or the magnitude of steel stress, or some relationship between the average strains in the reinforcement and in the concrete must be determined empirically. In this investigation it was chosen to evaluate k_s empirically. Using the parameter $k_1 k_3$ as determined from flexural failures, it was shown that there existed a simple relation between k_s and k . This shows why the previous attempt to use k as a measure of k_s gave satisfactory results.

The effect of web reinforcement was investigated next. It was found that a given amount of web reinforcement increased the shear strength of a beam in proportion to its strength without web reinforcement rather than by an amount determined solely by the physical properties of the web reinforcement. An empirical equation was found to give good correlation with test results. This equation was based on the test results for 80 beams. The average ratio between the measured and calculated moments was 1.012 and the standard deviation 0.085. This equation was found to be applicable for all angles of inclination and for different values of yield strength of web reinforcement. It was also found that there was no noticeable difference between the effectiveness of bent-up bars and stirrups serving as web reinforcement.

The relationship between shear-compression and flexural failures was investigated next. It was found that the amount of web reinforcement necessary to prevent shear failures could be determined for any beam by the shear-compression equations together with an expression for the ultimate flexural moment of the beam.

SIMPLE-SPAN T-BEAMS UNDER ONE OR TWO SYMMETRICAL CONCENTRATED LOADS

Since the moment-rotation relationship of a T-beam differs from that of a rectangular beam, the basic shear-compression equation was modified to apply for T-beams. This was done by a shape-factor which considered the differences in the moment of inertia for the two types of beams. The modified equation gave satisfactory agreement with test results whenever beams with very thin flanges relative to their depth and very wide flanges relative to their width of web were excluded. The effect of web reinforcement could be taken into consideration by the same expression as that for rectangular beams.

RESTRAINED BEAMS UNDER SYMMETRICAL CONCENTRATED LOADS

Simple-span beams under one or two symmetrical concentrated loads develop just one main diagonal crack at the location of the applied load and fail at that section. In restrained beams, shear and moment conditions are such as to permit the formation of several diagonal cracks. The beam may fail at any of these cracks, depending on the magnitudes of moment and shear and the arrangement of both longitudinal and web reinforcement.

It was found that whenever the possibility of local bond failures was precluded, the shear strength of a restrained beam was determined by the same equations as that of a simple-span beam. The critical section was the section of maximum moment.

Restrained beams with continuous top and bottom reinforcement may have another mode of failure. Under certain conditions, e.g. when the distance between a support and a load is short relative to the effective depth of the beam, a local bond failure may take place in the high bond-stress region between the sections of positive and negative moments. As the result of local destruction of bond, both the top and bottom longitudinal reinforcement are in tension at a certain section. This redistribution of the internal forces results in a greatly reduced shear strength of the beam.

The use of the quantity k_s as determined in the analysis of simple-span beams permitted the establishment of an express-

ion for the shear strength of a beam failing in shear after a redistribution of the internal forces. This equation was checked against test results and satisfactory agreement was obtained. The contribution of web reinforcement could be taken into consideration by the same equation as that for rectangular and T-beams.

LIMITATIONS OF SHEAR-COMPRESSION

All shear-compression equations were derived and checked for beams for which the compressive force-shear ratio, $a/d = M/Vd$, varied between 1.17 and 4.80. Within these limits, the shear strength of a beam was found to be determined solely by the physical properties of the beam. It was not a function of either the magnitude of shear or the moment-shear ratio at the section of failure.

However, as the M/Vd - ratio increases, the relative importance of shear in connection with the diagonal tension stresses decreases. Consequently, the extent of diagonal cracking is less pronounced and it was found that the shear strength of such beams was larger than that given by the shear-compression equations. Furthermore, the location of shear failure was not necessarily the section of maximum moment. For sufficiently large values of M/Vd , the beams failed in flexure rather than in shear. However, the upper limit of M/Vd for the applicability of shear-compression equations and the shear strength of a beam in the transition region between shear and flexural failures could not be determined quantitatively because of insufficient experimental data.

Conversely, for very small values of a/d , the beams did not fail in shear-compression. The mode of failure appeared to be an actual shearing off of the compression zone of the concrete. This type of failure was tentatively called shear-proper. It was also found that the shear strength of such beams depended on the x/D -ratio rather than a/d ratio, where x denotes the clear distance between two load bearing blocks and D the total depth of beam. For x/D equal to zero, the shear strength of a beam could be related to a nominal shearing stress. An entirely empirical equation was found to give satisfactory agreement with test results. As the ratio x/D increased, the ratio between the measured and calculated shearing stresses decreased. Since

the number of tests was limited, no expression could be determined for the variation of this ratio with the parameter x/D . The transition region between shear-proper and shear-compression was estimated to lie approximately between x/D equal to 0.8 and 1.1, increasing as the amount of vertical stirrups increased.

DISTRIBUTED LOADING

For simple-span beams under uniform loading, the value of M/Vd ranges from zero at the section of no moment to infinity at the section of maximum moment. Since there are no diagonal cracks in the region of maximum moment and the inclination of cracks is very small for high values of M/Vd , the beam cannot fail in shear unless, at a critical section, a section other than that of maximum moment, the value of M/Vd is such as to permit sufficient diagonal cracking and the moment itself is large enough to produce a shear-compression failure. From the test results studied, this critical section was set tentatively at M/Vd about 4.5. However, since only a few T-beams were tested under conditions which simulated uniform loading, the validity of the above concept of shear failures and a more reliable value of the critical M/Vd must be established by a more comprehensive test program.

It is conceivable that the same procedure can be used for any type of beam under either uniform or distributed loading to determine its strength in shear. It involves only the determination of critical sections for shear failures. Provided that the value of M/Vd is in the region of shear-compression, the shear-compression equations can be used directly at sections where maximum shear and maximum moment coincide. In regions of maximum moment and no shear the critical section at which the shear-compression equations should be used is given by the critical value of M/Vd .

VITA

Armas Laupa was born on October 1, 1925, in Lehtse, Estonia. He attended several secondary schools in Estonia, served one year in the Finnish Navy from which he was honorably discharged in September 1944, and graduated from Sigtuna Estonian Gymnasium in Sweden in February, 1947. In the academic year of 1947-48 he attended the Royal Institute of Technology in Stockholm, Sweden. He then was awarded a scholarship by the Rotary Clubs of Georgia and entered the Georgia Institute of Technology where he received the degree of Bachelor of Civil Engineering, with highest honors, in September 1950. He continued his studies at the University of Illinois where the degree of Master of Science in Civil Engineering was conferred upon him in June 1952.

At different times he has been employed with Hakon Alberg, Architect, SAR, in Stockholm, Sweden; with Civ.-Ing. Sven Tyren, Consulting Engineer in Stockholm, Sweden; with the Engineering Department, Colonial Stores Inc. in Atlanta, Georgia, and with the Crosby Construction Company in Chicago, Illinois. At the University of Illinois he held a research assistantship in Theoretical and Applied Mechanics from September 1950 to June 1952 and in Civil Engineering from June 1952 to June 1953. At this time he was appointed a research associate in Civil Engineering. He has been assigned to various cooperative investigations of reinforced concrete slabs and beams.

He is a junior member of the American Society of Civil Engineers, student member of the American Concrete Institute, associate member of the Society of Sigma Xi, and a member of Tau Beta Pi and Chi Epsilon. He was awarded the General Lucius Clay Scholarship Award at the Georgia Institute of Technology.

